

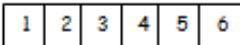




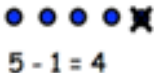

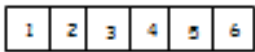
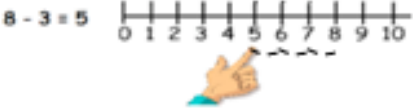
Addition

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

GUIDANCE / MODELS AND IMAGES	KEY VOCABULARY
<p>If available, Numicon shapes are introduced straight away and can be used to:</p> <ul style="list-style-type: none"> • identify 1 more/less • combine pieces to add. • find number bonds. • add without counting. <p>Children can record this by printing or drawing around Numicon pieces.</p> <p>Children begin to combine groups of objects using concrete apparatus</p>  <p>Construct number sentences verbally or using cards to go with practical activities.</p> <p>Children are encouraged to read number sentences aloud in different ways "Three add two equals 5" "5 is equal to three and two"</p> <p>Children make a record in pictures, words or symbols of addition activities already carried out.</p> <p>Solve simple problems using fingers</p>  $5 + 1 = 6$ <p>Number tracks can be introduced to count up on and to find one more:</p>  <p>What is 1 more than 4? 1 more than 13?</p> <p>Number lines can then be used alongside number tracks and practical apparatus to solve addition calculations and word problems.</p>  <p>Children will need opportunities to look at and talk about different models and images as they move between representations.</p>	<p>Games and songs can be a useful way to begin using vocabulary involved in addition e.g. Alice the Camel</p> <p>add</p> <p>more</p> <p>and</p> <p>make</p> <p>sum</p> <p>total</p> <p>altogether</p> <p>score</p> <p>double</p> <p>one more, two more, ten more...</p> <p>how many more to make...?</p> <p>how many more is... than...?</p>





Subtraction

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

GUIDANCE / MODELS AND IMAGES	KEY VOCABULARY
<p>Children begin with mostly pictorial representations</p> <div data-bbox="202 411 436 468">  </div> <p>Concrete apparatus is used to relate subtraction to taking away and counting how many objects are left.</p> <p>Concrete apparatus models the subtraction of 2 objects from a set of 5.</p> <div data-bbox="1302 525 1456 596">  </div> <p>Construct number sentences verbally or using cards to go with practical activities.</p> <p>Children are encouraged to read number sentences aloud in different ways "five subtract one leaves four" "four is equal to five subtract one"</p> <p>Children make a record in pictures, words or symbols of subtraction activities already carried out.</p> <p>Solve simple problems using fingers</p> <div data-bbox="654 919 857 1012">  </div> <p>Number tracks can be introduced to count back and to find one less:</p> <div data-bbox="989 1005 1244 1055">  </div> <p>What is 1 less than 9? 1 less than 20?</p> <p>Number lines can then be used alongside number tracks and practical apparatus to solve subtraction calculations and word problems. Children count back under the number line.</p> <div data-bbox="1051 1176 1466 1283">  </div> <p>Children will need opportunities to look at and talk about different models and images as they move between representations.</p>	<p>Games and songs can be a useful way to begin using vocabulary involved in subtraction e.g. Five little men in a flying saucer</p> <p>take (away)</p> <p>leave</p> <p>how many are left/left over?</p> <p>how many have gone?</p> <p>one less, two less... ten less...</p> <p>how many fewer is... than...?</p> <p>difference between</p> <p>is the same as</p>



Multiplication

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.


GUIDANCE / MODELS AND IMAGES	KEY VOCABULARY
<p>The link between addition and multiplication can be introduced through doubling.</p> <p>If available, Numicon is used to visualise the repeated adding of the same number. These can then be drawn around or printed as a way of recording.</p> <p>Children begin with mostly pictorial representations:</p> <div data-bbox="185 554 568 636">  </div> <p>How many groups of 2 are there?</p> <p>Real life contexts and use of practical equipment to <u>count in repeated groups of the same size</u>:</p> <div data-bbox="191 791 724 886">  </div> <p>How many wheels are there altogether?</p> <div data-bbox="877 805 1425 911">  </div> <p>How much money do I have?</p> <div data-bbox="803 1005 954 1096">  </div> <p>Count in twos; fives; tens both aloud and with objects</p> <p>Children are <u>given multiplication problems set in a real life context</u>. Children are encouraged to visualise the problem.</p> <p>How many fingers on two hands? How many sides on three triangles? How many legs on four ducks?</p> <p>Children are encouraged to read number sentences aloud in different ways "five times two makes ten" "ten is equal to five multiplied by two"</p>	<p>lots of</p> <p>groups of</p> <p>times</p> <p>multiply</p> <p>multiplied by</p> <p>multiple of</p> <p>once, twice, three times... ten times...</p> <p>...times as (big, long, wide... and so on)</p> <p>repeated addition</p> <p>double</p>

Division and fractions

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

GUIDANCE / MODELS AND IMAGES	KEY VOCABULARY
<p>The ELG states that children solve problems, including doubling, halving and sharing.</p> <p>Children need to see and hear representations of division as both grouping and sharing.</p> <p>Division can be introduced through halving.</p> <p>Children begin with mostly pictorial representations linked to real life contexts:</p> <div style="display: flex; align-items: flex-start;"> <div style="margin-right: 10px;">  </div> <div> <p>Grouping model Mum has 6 socks. She grouped them into pairs – how many pairs did she make?</p> </div> </div> <div style="display: flex; align-items: flex-start; margin-top: 10px;"> <div style="margin-right: 10px;">  </div> <div> <p>Sharing model I have 10 sweets. I want to share them with my friend. How many will we have each?</p> </div> </div> <p>Children have a go at recording the calculation that has been carried out.</p>	<p>halve</p> <p>share, share equally</p> <p>one each, two each, three each...</p> <p>group in pairs, threes...</p> <p>tens</p> <p>equal groups of</p> <p>divide</p> <p>divided by</p> <p>divided into</p> <p>left, left over</p>

FRACTIONS

GUIDANCE / MODELS AND IMAGES	KEY VOCABULARY
<p>Although not explicit in the Development Matters document, the sharing model is a useful way of introducing young children to fractions and calculating with fractions.</p> <p>Setting the problems in real life context and solving them with <u>concrete apparatus</u> will support children's understanding.</p> <p>"I have got 5 bones to share between my two dogs. How many bones will they get each?"</p> <div style="display: flex; align-items: center; justify-content: center; margin-top: 10px;">  </div> <p>Children have a go at recording the calculation that has been carried out.</p> $2\frac{1}{2} + 2\frac{1}{2} = 5$	<p>As division vocabulary plus:</p> <p>fraction</p> <p>half</p> <p>halves</p> <p>third</p> <p>thirds</p>

Addition Year 1

+ = signs and missing numbers

Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.

$$2 = 1 + 1$$

$$2 + 3 = 4 + 1$$

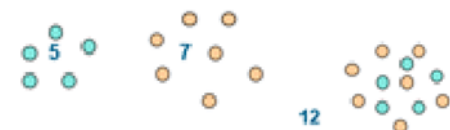
Missing numbers need to be placed in all possible places.

$$3 + 4 = \square \quad \square = 3 + 4$$

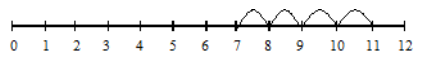
$$3 + \square = 7 \quad 7 = \square + 4$$

Counting and Combining sets of Objects

Combining two sets of objects (aggregation) which will progress onto adding on to a set (augmentation)



Understanding of counting on with a numbertrack.



Understanding of counting on with a numberline (supported by models and images).

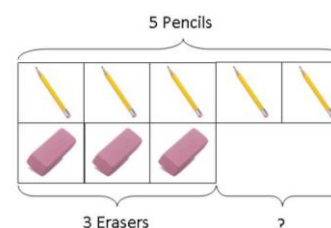
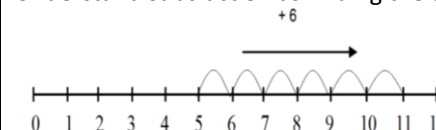
Subtraction Year 1

Missing number problems e.g. $7 = \square - 9$; $20 - \square = 9$; $15 - 9 = \square$; $\square - \square = 11$; $16 - 0 = \square$

Use concrete objects and pictorial representations. If appropriate, progress from using number lines with every number shown to number lines with significant numbers shown. Understand subtraction as take-away (removing items from a set):



Understand subtraction as finding the difference (comparing 2 sets):



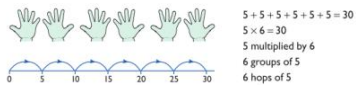
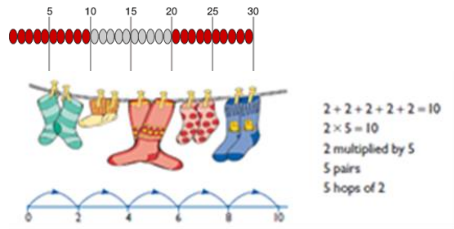
The above model would be introduced with concrete objects which children can move (including cards with pictures) before progressing to pictorial representation.

The use of other images is also valuable for modelling subtraction e.g. Numicon, bundles of straws, Dienes apparatus, multi-link cubes, bead strings

Multiplication Year 1

Understand multiplication is related to doubling and combining groups of the same size (repeated addition)

Washing line, and other practical resources for counting. Concrete objects. Numicon; bundles of straws, bead strings

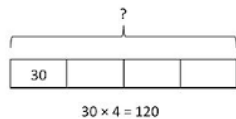


Problem solving with concrete objects (including money and measures)

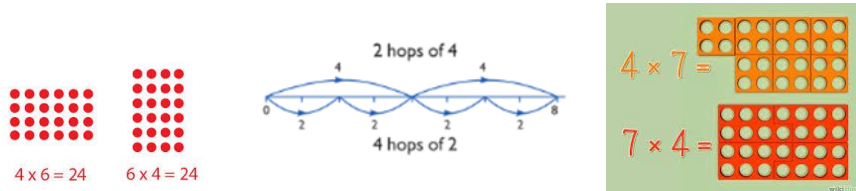
Use cuisenaire and bar method to develop the vocabulary relating to 'times' –

Pick up five, 4 times / Pick up four, 5 times

5 x 4 = 5 groups of 4 or 4 groups of 5



Use arrays to understand multiplication can be done in any order (commutative)



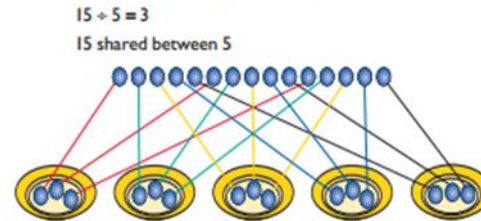
Division Year 1

Children must have secure counting skills- being able to confidently count in 2s, 5s and 10s. Children should be given opportunities to reason about what they notice in number patterns.

Group AND share small quantities- understanding the difference between the two concepts.

Sharing

Develops importance of one-to-one correspondence.



Children should be taught to share using concrete apparatus.

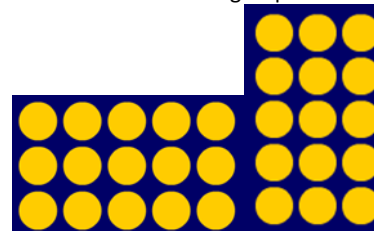
Grouping

Children should apply their counting skills to develop some understanding of grouping.



Use of arrays as a pictorial representation for division. 15 ÷ 3 = 5 There are 5 groups of 3.

15 ÷ 5 = 3 There are 3 groups of 5.



Children should be able to find ½ and ¼ and simple fractions of objects, numbers and quantities.

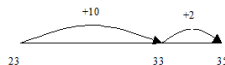
Addition Year 2

Missing number problems e.g. $14 + 5 = 10 + \square$ $32 + \square + \square = 100$ $35 = 1 + \square + 5$

It is valuable to use a range of representations (also see Y1). Continue to use number lines to develop understanding of:

Counting on in tens and ones

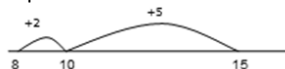
$$\begin{aligned} 23 + 12 &= 23 + 10 + 2 \\ &= 33 + 2 \\ &= 35 \end{aligned}$$



Partitioning and bridging through 10.

The steps in addition often bridge through a multiple of 10
e.g. Children should be able to partition the 7 to relate adding the 2 and then the 5.

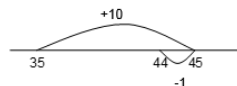
$$8 + 7 = 15$$



Adding 9 or 11 by adding 10 and adjusting by 1

e.g. Add 9 by adding 10 and adjusting by 1

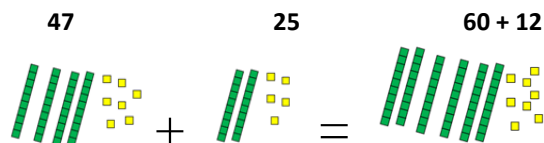
$$35 + 9 = 44$$



Towards a Written Method

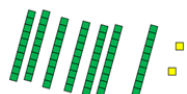
Partitioning in different ways and recombine

$$47 + 25$$



Leading to exchanging:

$$72$$



Expanded written method

$$\begin{array}{r} 40 + 7 \\ + 20 + 5 \\ \hline 60 + 12 = 72 \end{array}$$

$$40 + 7 + 20 + 5 =$$

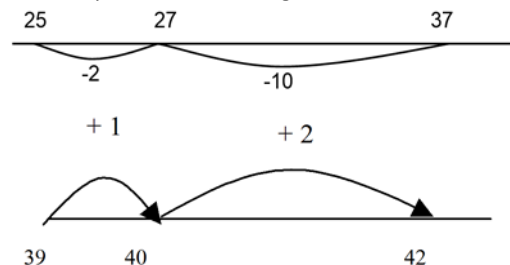
$$40 + 20 + 7 + 5 =$$

$$60 + 12 = 72$$

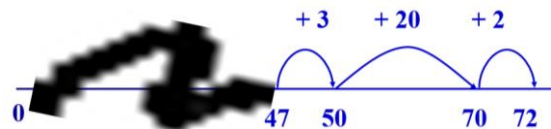
Subtraction Year 2

Missing number problems e.g. $52 - 8 = \square$; $\square - 20 = 25$; $22 = \square - 21$; $6 + \square + 3 = 11$

It is valuable to use a range of representations (also see Y1). Continue to use number lines to model take-away and difference. E.g.



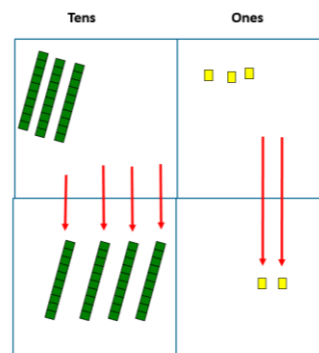
The link between the two may be supported by an image like this, with 47 being taken away from 72, leaving the difference, which is 25.



The bar model should continue to be used, as well as images in the context of **measures**.

Towards written methods

Recording addition and subtraction in expanded columns can support understanding of the quantity aspect of place value and prepare for efficient written methods with larger numbers. The numbers may be represented with Dienes apparatus. E.g. $75 - 42$



$$\begin{array}{r} 70 \ 5 \\ - 40 \ 2 \\ \hline 30 \ 3 \end{array}$$

Multiplication Year 2

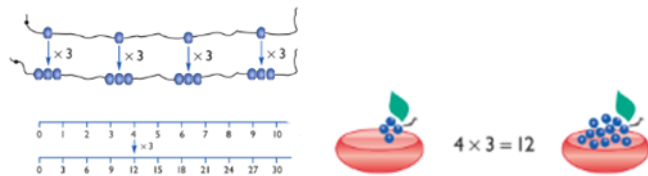
Expressing multiplication as a number sentence using x

Using understanding of the inverse and practical resources to solve missing number problems.

$$\begin{array}{ll} 7 \times 2 = \square & \square = 2 \times 7 \\ 7 \times \square = 14 & 14 = \square \times 7 \\ \square \times 2 = 14 & 14 = 2 \times \square \\ \square \times \bigcirc = 14 & 14 = \square \times \bigcirc \end{array}$$

Develop understanding of multiplication using array and number lines (see Year 1). Include multiplications not in the 2, 5 or 10 times tables.

Begin to develop understanding of multiplication as scaling (3 times bigger/taller)



Doubling numbers up to $10 + 10$



Link with understanding scaling Using known doubles to work out double 2d numbers (double 15 = double 10 + double 5)

Towards written methods

Use jottings to develop an understanding of doubling two digit numbers.

$$\begin{array}{r} 16 \\ 10 \quad 6 \\ \times 2 \quad \times 2 \\ \hline 20 \quad 12 \end{array}$$

Division Year 2

\div = signs and missing numbers

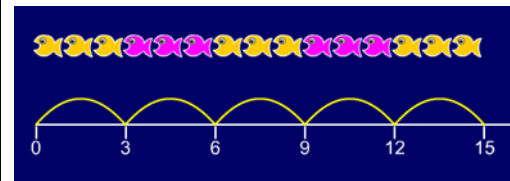
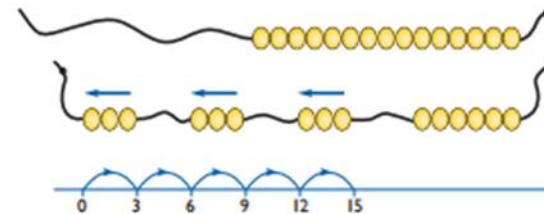
$$\begin{array}{ll} 6 \div 2 = \square & \square = 6 \div 2 \\ 6 \div \square = 3 & 3 = 6 \div \square \\ \square \div 2 = 3 & 3 = \square \div 2 \\ \square \div \nabla = 3 & 3 = \square \div \nabla \end{array}$$

Know and understand sharing and grouping- introducing children to the \div sign.

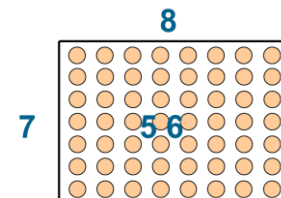
Children should continue to use grouping and sharing for division using practical apparatus, arrays and pictorial representations.

Grouping using a numberline

Group from zero in jumps of the divisor to find our 'how many groups of 3 are there in 15?'.
 $15 \div 3 = 5$



Continue work on arrays. Support children to understand how multiplication and division are inverse. Look at an array – what do you see?



Addition Year 3

Missing number problems using a range of equations as in Year 1 and 2 but with appropriate, larger numbers.

Partition into tens and ones

Partition both numbers and recombine.

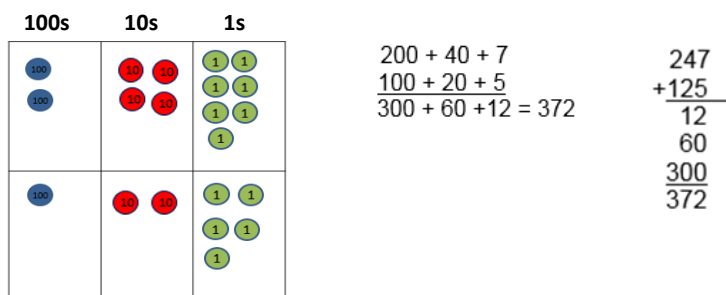
Count on by partitioning the second number only e.g.

$$\begin{aligned} 247 + 125 &= 247 + 100 + 20 + 5 \\ &= 347 + 20 + 5 \\ &= 367 + 5 \\ &= 372 \end{aligned}$$

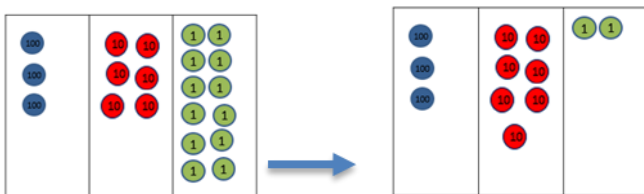
Children need to be secure adding multiples of 100 and 10 to any three-digit number including those that are not multiples of 10.

Towards a Written Method

Introduce expanded column addition modelled with place value counters (Dienes could be used for those who need a less abstract representation)



Leading to children understanding the exchange between tens and ones.



Some children may begin to use a formal columnar algorithm, initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method.

$$\begin{array}{r} 247 \\ +125 \\ \hline 372 \\ 10 \end{array}$$

Subtraction Year 3

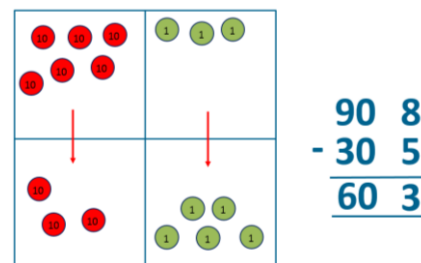
Missing number problems e.g. $\square = 43 - 27$; $145 - \square = 138$; $274 - 30 = \square$; $245 - \square = 195$; $532 - 200 = \square$; $364 - 153 = \square$

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving (see Y1 and Y2).

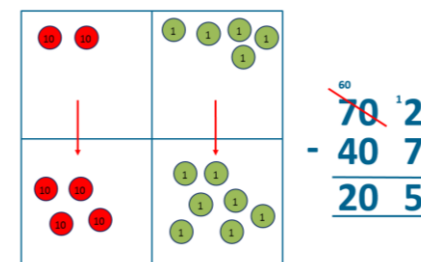
Children should make choices about whether to use complementary addition or counting back, depending on the numbers involved.

Written methods (progressing to 3-digits)

Introduce expanded column subtraction with no decomposition, modelled with place value counters (Dienes could be used for those who need a less abstract representation)



For some children this will lead to exchanging, modelled using place value counters (or Dienes).



A number line and expanded column method may be compared next to each other.

Some children may begin to use a formal columnar algorithm, initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method.

Multiplication Year 3

Missing number problems

Continue with a range of equations as in Year 2 but with appropriate numbers.

Mental methods

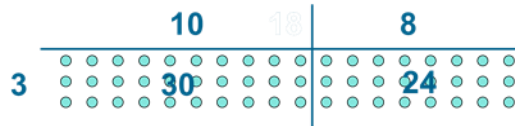
Doubling 2 digit numbers using partitioning

Demonstrating multiplication on a number line – jumping in larger groups of amounts

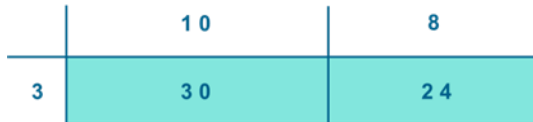
$$13 \times 4 = 10 \text{ groups } 4 = 3 \text{ groups of } 4$$

Written methods (progressing to 2d x 1d)

Developing written methods using understanding of visual images



Develop onto the grid method



Give children opportunities for children to explore this and deepen understanding using Dienes apparatus and place value counters

Division Year 3

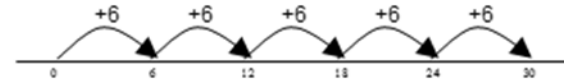
\div = signs and missing numbers

Continue using a range of equations as in year 2 but with appropriate numbers.

Grouping

How many 6's are in 30?

$30 \div 6$ can be modelled as:



Becoming more efficient using a numberline

Children need to be able to partition the dividend in different ways.

$$48 \div 4 = 12$$



Remainders

$$49 \div 4 = 12 \text{ r}1$$



Sharing – 49 shared between 4. How many left over?

Grouping – How many 4s make 49. How many are left over?

Place value counters can be used to support children apply their knowledge of grouping.

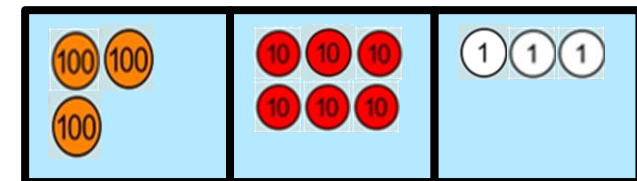
For example:

$$60 \div 10 = \text{How many groups of 10 in 60?}$$

$$600 \div 100 = \text{How many groups of 100 in 600?}$$

$$363 \div 3 =$$

$$\begin{array}{r} 121 \\ 3 \overline{) 363} \end{array}$$



Addition Year 4

Missing number/digit problems:

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

Written methods (progressing to 4-digits)

Expanded column addition modelled with place value counters, progressing to calculations with 4-digit numbers.

			$\begin{array}{r} 200 + 40 + 7 \\ 100 + 20 + 5 \\ 300 + 60 + 12 = 372 \end{array}$
			$\begin{array}{r} 247 \\ + 125 \\ \hline 12 \\ 60 \\ 300 \\ 372 \end{array}$

Compact written method

Extend to numbers with at least four digits.

7	1	5	1	

$$\begin{array}{r} 2634 \\ + 4517 \\ \hline 7151 \\ \hline \end{array}$$

Children should be able to make the choice of reverting to expanded methods if experiencing any difficulty.

Extend to up to two places of decimals (same number of decimal places) and adding several numbers (with different numbers of digits).

$$\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \end{array}$$

1 1

hundreds	ten	ones	tenths	hundredths
			.	
			.	
			.	

Subtraction Year 4

Missing number/digit problems: $456 + \square = 710$;

$1\square7 + 6\square = 200$; $60 + 99 + \square = 340$; $200 - 90 - 80 = \square$; $225 - \square = 150$; $\square - 25 = 67$; $3450 - 1000 = \square$; $\square - 2000 = 900$

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

Written methods (progressing to 4-digits)

Expanded column subtraction with decomposition, modelled with place value counters, progressing to calculations with 4-digit numbers.

			$\begin{array}{r} 200 \overset{20}{\cancel{30}} \overset{1}{2} \\ - 100 \overset{10}{\cancel{10}} \overset{4}{4} \\ \hline 100 \overset{10}{\cancel{10}} \overset{8}{8} \end{array}$
			$\begin{array}{r} 200 \overset{20}{\cancel{30}} \overset{1}{2} \\ - 100 \overset{10}{\cancel{10}} \overset{4}{4} \\ \hline 100 \overset{10}{\cancel{10}} \overset{8}{8} \end{array}$

If understanding of the expanded method is secure, children will move on to the formal method of decomposition, which again can be initially modelled with place value counters.

			$\begin{array}{r} 232 \overset{2}{\cancel{3}} \overset{1}{2} \\ - 114 \\ \hline 118 \end{array}$
			$\begin{array}{r} 232 \overset{2}{\cancel{3}} \overset{1}{2} \\ - 114 \\ \hline 118 \end{array}$

Multiplication Year 4

Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits

$$\square 2 \times 5 = 160$$

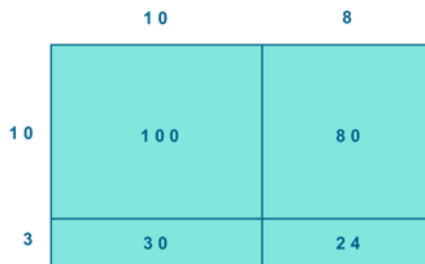
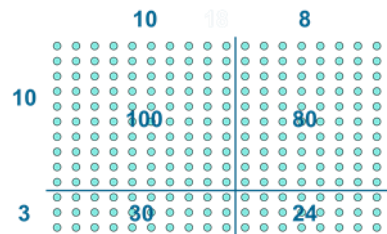
Mental methods

Counting in multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.

Solving practical problems where children need to scale up. Relate to known number facts. (e.g. how tall would a 25cm sunflower be if it grew 6 times taller?)

Written methods (progressing to 3d x 2d)

Children to embed and deepen their understanding of the grid method to multiply up 2d x 2d. Ensure this is still linked back to their understanding of arrays and place value counters.



Division Year 4

÷ = signs and missing numbers

Continue using a range of equations as in year 3 but with appropriate numbers.

Sharing, Grouping and using a number line

Children will continue to explore division as sharing and grouping, and to represent calculations on a number line until they have a secure understanding. Children should progress in their use of written division calculations:

*Using tables facts with which they are fluent

*Experiencing a logical progression in the numbers they use, for example:

1) Dividend just over 10x the divisor, e.g. $84 \div 7$

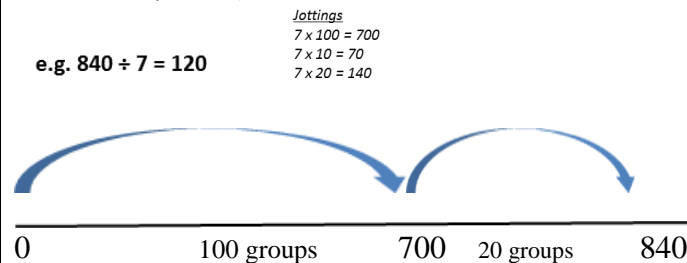
2) Dividend just over 10x the divisor when the divisor is a teen number, e.g. $173 \div 15$ (learning sensible strategies for calculations such as $102 \div 17$)

3) Dividend over 100x the divisor, e.g. $840 \div 7$

4) Dividend over 20x the divisor, e.g. $168 \div 7$

All of the above stages should include calculations with remainders as well as without.

Remainders should be interpreted according to the context. (i.e. rounded up or down to relate to the answer to the problem)

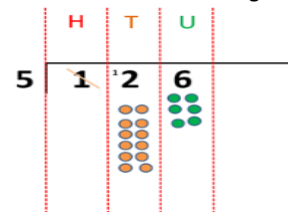


Formal Written Methods

Formal short division should only be introduced once children have a good understanding of division, its links with multiplication and the idea of 'chunking up' to find a target number (see use of number lines above)

Short division to be modelled for understanding using place value counters as shown below.

Calculations with 2 and 3-digit dividends. E.g. fig 1



Addition Year 5

Missing number/digit problems:

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. Children should practise with increasingly large numbers to aid fluency
e.g. $12462 + 2300 = 14762$

Written methods (progressing to more than 4-digits)

As year 4, progressing when understanding of the expanded method is secure, children will move on to the formal columnar method for whole numbers and decimal numbers as an efficient written algorithm.

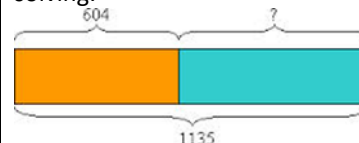
$$\begin{array}{r} 172.83 \\ + 54.68 \\ \hline 227.51 \\ 1 \ 1 \end{array}$$

Place value counters can be used alongside the columnar method to develop understanding of addition with decimal numbers.

Subtraction Year 5

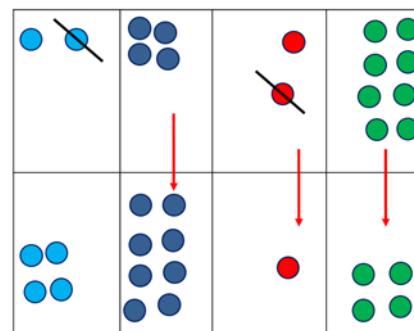
Missing number/digit problems: $6.45 = 6 + 0.4 + \square$; $119 - \square = 86$; $1\ 000\ 000 - \square = 999\ 000$; $600\ 000 + \square + 1000 = 671\ 000$; $12\ 462 - 2\ 300 = \square$

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.



Written methods (progressing to more than 4-digits)

When understanding of the expanded method is secure, children will move on to the formal method of decomposition, which can be initially modelled with place value counters.



$$\begin{array}{r} \overset{5}{\cancel{6}} \overset{1}{\cancel{2}} \overset{2}{\cancel{3}} \overset{1}{\cancel{2}} \\ - 4814 \\ \hline 1418 \end{array}$$

Progress to calculating with decimals, including those with different numbers of decimal places.

Multiplication Year 5

Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits

Mental methods

X by 10, 100, 1000 using moving digits ITP

Use practical resources and jottings to explore equivalent statements (e.g. $4 \times 35 = 2 \times 2 \times 35$)

Recall of prime numbers up to 19 and identify prime numbers up to 100 (with reasoning)

Solving practical problems where children need to scale up. Relate to known number facts.

Identify factor pairs for numbers

Written methods (progressing to 4d x 2d)

Long multiplication using place value counters

Children to explore how the grid method supports an understanding of long multiplication (for 2d x 2d)

	10	8
10	100	80
3	30	24

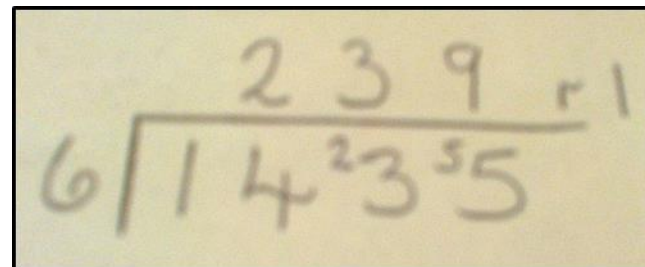
		1	8		
	×	1	3		
	1	8	0		
		5	4		
	2	3	4		

Division Year 5

Formal Written Methods

Continued as shown in Year 4, leading to the efficient use of a formal method. The language of grouping to be used (see link from fig. 1 in Year 4)

E.g. $1435 \div 6$



Children begin to practically develop their understanding of how express the remainder as a decimal or a fraction. Ensure practical understanding allows children to work through this (e.g. what could I do with this remaining 1? How could I share this between 6 as well?)